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SEPARATION IMPACT ON A PLATE FLOATING ON THE SURFACE OF AN IDEAL INCOMPRESSIBLE FLUID IN A BOUNDED TANK

M. V. Norkin

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The paper studies the planar problem of separation impact on a plate floating on the surface of an ideal incompressible fluid in a bounded tank. The problem is solved using an asymptotic method under the assumption that the immovable rigid walls of the tank are at a large distance from the plate. It is concluded that the tank walls of arbitrary shape have an ambiguous effect on the fluid particle separation zone formed on the plate surface is revealed. Examples of solutions are given.

Key words: *ideal fluid, impact, floating body, bounded tank, separation zone, velocity potential, asymptotic expressions.*

The problem of hydrodynamic separation impact was formulated by Sedov [1], who developed methods for calculating the pulsed loads acting on a body and fluid motions using the theory of functions of a complex variable. Using the methods described in [1], analytical solutions were derived for several specific cases [1–4]. The planar problem of separation impact on floating bodies was numerically solved in [5]. In all studies of separation impact, the fluid was assumed to be infinite.

In the present paper, we study the planar problem of separation impact on a plate floating on the surface of an ideal incompressible fluid in a bounded tank. A solution of the problem is derived using an asymptotic method based on the assumption that the immovable rigid tank walls are at a large distance from the plate. Previously, this approach has been used to solve the spatial problem of separation-free impact on floating bodies [6–8].

1. Formulation of the Problem. We consider the planar problem of vertical impact on a plate floating on the surface of an ideal incompressible fluid in a bounded tank. Upon impact, the fluid particles are assumed to separate from the plate surface (separation impact). Let the body and the fluid be at rest before the impact. Then, after the impact, the fluid motion is potential and the velocity potential Φ of the fluid particles induced by the impact is determined by solving a mixed boundary-value problem of potential theory with an *a priori* unknown contact area [1]:

$$\Delta\Phi = 0, \qquad r \in D; \tag{1.1}$$

$$\Phi \leqslant 0, \qquad \frac{\partial\Phi}{\partial y} = V_n, \qquad r \in S_{11}; \tag{1.2}$$

$$\Phi = 0, \qquad \frac{\partial\Phi}{\partial y} \geqslant V_n, \qquad r \in S_{12}; \tag{1.3}$$

$$\Phi = 0, \qquad r \in S_2; \tag{1.4}$$

$$\frac{\partial\Phi}{\partial n} = 0, \qquad r \in S_3. \tag{1.5}$$

Here D is the region occupied by the fluid, $V_n = v_0 - \omega x$, $r = (x, y)$, v_0 and $\omega > 0$ are the translational and angular velocities of the plate induced by the impact, respectively, $S_1 = S_{11} \cup S_{12}$ is the plate surface, $S_{11} = \{y = 0,$

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$-a < x < c\}$ is the part of the surface on which there is no separation of fluid particles, $S_{12} = \{y = 0,\, c < x < a\}$ is the separation zone, S_2 is the free surface of the fluid, and S_3 is the immovable rigid tank boundary. The pulse pressure is $P_t = -\rho\Phi$ (ρ is the fluid density). The Cartesian coordinates x and y are introduced such that the x axis runs along the free-surface line, the y axis is directed vertically inward (into the fluid depth), and the coordinate origin is at the center of the plate.

We assume that the boundary S_3 is obtained by a homothetic transformation with center at the coordinate origin and the coefficient h of a certain fixed surface S_3^0: $S_3 = hS_3^0$ ($x = hx^0$, $y = hy^0$).

Below, the following notation is used: D^0 is the internal region with the boundaries $y = 0$ and S_3^0, G is the half-plane $y > 0$, and Φ_1 and c_∞ are the velocity potential and separation point for $h = \infty$, respectively.

In constructing asymptotic expressions for large h, boundary conditions (1.2) and (1.3) are conveniently transformed to the following restrictions ($r \in S_1$):

$$\frac{\partial\Phi}{\partial y} - V_n \geqslant 0; \tag{1.6}$$

$$\Phi\Big(\frac{\partial\Phi}{\partial y} - V_n\Big) = 0; \tag{1.7}$$

$$\Phi \leqslant 0. \tag{1.8}$$

Relations (1.6)–(1.8) can be rewritten as one-side limitations for the new function $-\Phi$. In this case, the derivative of the function $-\Phi$ is calculated with respect to the outward normal to the region D. In this case, the problem is formulated as a variational inequality that yields the theorem of existence and uniqueness of this problem [9]. Thus, for a bounded region in Sobolev space $H^1(D)$ there is a unique solution of the problem of hydrodynamic separation impact. We note that these issues for the problem of penetration of a rigid body into water in a similar mathematical formulation were studied in [10].

To complete the formulation of the problem, it is necessary to separately write the momentum equation and the angular momentum equation for the plate under impact. These equations define the relationship between the external impact momentum and its point, on the one hand, and the impact-induced translational and angular velocities of the plate, on the other hand. If the mass and moment of inertia of the plate are ignored, these equations reduce to the relations

$$P_x = 0, \qquad I + P_y = 0, \qquad M - x_0 P_y = 0. \tag{1.9}$$

Here P_x and P_y are the components of the external impact momentum applied to the plate at the point $(x_0, 0)$ and I and M are the total impact momentum and the angular momentum with respect to the coordinate origin acting on the plate during the impact:

$$I = \rho \int_{-a}^{c} \Phi\, dx, \qquad M = -\rho \int_{-a}^{c} x\Phi\, dx. \tag{1.10}$$

From equalities (1.9), we find the coordinate of the momentum point

$$x_0 = -M/I. \tag{1.11}$$

2. Solution for an Unbounded Fluid. We first solve the problem of separation impact on a plate floating on the surface of an unbounded fluid ($h = \infty$). Placing the coordinate origin at the center of the segment $[-a, c]$, we arrive at the problem

$$\Delta u = 0, \qquad -\infty < x < \infty, \quad y > 0;$$

$$\frac{\partial u}{\partial y} = v_0 + \omega\,\frac{a-c}{2} - \omega x, \qquad y = 0, \quad |x| < \frac{a+c}{2}; \tag{2.1}$$

$$u = 0, \qquad y = 0, \quad |x| > \frac{a+c}{2}; \tag{2.2}$$

$$\frac{\partial u}{\partial y} \geqslant v_0 + \omega\,\frac{a-c}{2} - \omega x, \qquad y = 0, \quad \frac{a+c}{2} < x < \frac{3a-c}{2}; \tag{2.3}$$

$$u \leqslant 0, \qquad y = 0, \quad |x| < \frac{a+c}{2}; \tag{2.4}$$

$$u \to 0, \qquad x^2 + y^2 \to \infty, \tag{2.5}$$

with the function $u(x, y) = \Phi(x - (a - c)/2, y)$. A solution of the Laplace equation that satisfies conditions (2.1), (2.2), and (2.5) for any fixed $c \in [-a, a]$ is obtained by separating the variables in Cartesian coordinates using the method of pair integral equations. The solution is written in final form as

$$u = (v_0 + \omega(a - c)/2)u_1 + \omega u_2, \tag{2.6}$$

where

$$u_1(x, y) = \int\limits_0^\infty A(\lambda) \exp(-\lambda y) \cos \lambda x \, d\lambda, \qquad A(\lambda) = \int\limits_0^{(a+c)/2} \varphi(s) J_0(\lambda s) \, ds,$$

$$u_2(x, y) = \int\limits_0^\infty \lambda B(\lambda) \exp(-\lambda y) \sin \lambda x \, d\lambda, \qquad B(\lambda) = \int\limits_0^{(a+c)/2} \psi(s) J_0(\lambda s) \, ds,$$

$$\varphi(s) = -s, \qquad \psi(s) = -\frac{s^3}{4} + \frac{1}{4}\Big(\frac{a+c}{2}\Big)^2 s.$$

The constant c is chosen such that conditions (2.3) and (2.4) are satisfied. Substitution of function (2.6) into these equations yields the inequalities

$$v_0 + \omega \frac{a-c}{2} - \frac{\omega}{2} x \geqslant 0, \qquad x \in \Big[-\frac{a+c}{2}, \frac{a+c}{2}\Big],$$

$$\omega x^2 - \Big(v_0 + \omega \frac{a-c}{2}\Big)x - \frac{\omega}{2}\Big(\frac{a+c}{2}\Big)^2 \geqslant 0, \qquad x \in \Big[\frac{a+c}{2}, \frac{3a-c}{2}\Big].$$

The first inequality leads to the relation $c \leqslant a/3 + 4v_0/(3\omega)$, and the second inequality, leads to the relation $c \geqslant a/3 + 4v_0/(3\omega)$. As a result, to determine the separation point c_∞ and the velocity potential Φ_1 on the plate surface, we use the explicit relations

$$c_\infty = a/3 + 4v_0/(3\omega); \tag{2.7}$$

$$\Phi_1(x, 0) = -\sqrt{\Big(\frac{a+c_\infty}{2}\Big)^2 - \Big(x + \frac{a-c_\infty}{2}\Big)^2}\Big(v_0 + \frac{\omega}{2}\,\frac{a-c_\infty}{2} - \frac{\omega}{2}\,x\Big), \quad -a < x < c_\infty. \tag{2.8}$$

Determining the normal velocity component on the plate surface, it can be shown that it is continuous everywhere on $(-a, a)$.

Using Eqs. (1.10), (1.11), (2.7), and (2.8), we find the relationship between the momentum point x_0 and the separation point c_∞:

$$x_0 = -(5a - 3c_\infty)/8, \qquad c_\infty = (5a + 8x_0)/3. \tag{2.9}$$

Investigation of the problem of vertical separation-free impact of a plate showed that if the point of the external impact momentum P lies within $[-a/4, a/4]$, fluid particles do not separate from the plate surface. Otherwise, separation occurs. Hence, the point x_0 in formulas (2.9) must satisfy the inequalities $-a \leqslant x_0 < -a/4$.

3. Asymptotic Expressions for Large *h*. The velocity potential Φ defined by formulas (1.1), (1.4), (1.5), and (1.6)–(1.8) is sought for in the form of the series $\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \ldots$. As a first approximation of Φ_1, we use the solution of the problem of separation impact on a plate floating on the surface of an unbounded fluid ($h = \infty$). It is assumed that upon impact, the plate acquires the same translational and angular velocities as in the case of a bounded region. At large distances from the plate, the function Φ_1 can be expanded in the harmonic series

$$\Phi_1 = -\frac{c_1 y}{\pi(x^2 + y^2)} - \frac{2c_2 xy}{\pi(x^2 + y^2)^2} - \frac{c_3(3x^2 y - y^3)}{\pi(x^2 + y^2)^3} - \ldots, \tag{3.1}$$

where

$$c_1 = \frac{\omega\pi}{4}\Big(\frac{a+c_\infty}{2}\Big)^3, \qquad c_2 = -\frac{\omega\pi(5a - 3c_\infty)}{32}\Big(\frac{a+c_\infty}{2}\Big)^3,$$

$$c_3 = \frac{\omega\pi}{64}(7a^2 - 6ac_\infty + 3c_\infty^2)\Big(\frac{a+c_\infty}{2}\Big)^3.$$

To eliminate the errors caused by the potential Φ_1 on the immovable boundary S_3, we consider this problem for a bounded tank in the absence of the plate:

$$\Delta\Phi_2 = 0, \quad r \in D, \quad (\Phi_2)_{y=0} = 0, \quad \Big(\frac{\partial \Phi_2}{\partial n}\Big)_{S_3} = \frac{c_1}{\pi} Q_1 + \frac{2c_2}{\pi} Q_2 + \frac{c_3}{\pi} Q_3, \tag{3.2}$$

where

$$Q_1 = \frac{\partial}{\partial n} \frac{y}{x^2+y^2}, \qquad Q_2 = \frac{\partial}{\partial n} \frac{xy}{(x^2+y^2)^2}, \qquad Q_3 = \frac{\partial}{\partial n} \frac{3x^2y-y^3}{(x^2+y^2)^3}.$$

In this case, we can confine ourselves to the first three terms of series (3.1). The contribution of the remaining terms to the asymptotic form of the potential Φ on S_1 is on the order of $O(h^{-5})$ as $h \to \infty$.

After the variable substitution $x \to hx$ and $y \to hy$ in (3.2), the function $f = f(x,y) = \Phi_2(hx, hy)$ is written as

$$f = \frac{c_1}{\pi} f_1 h^{-1} + \frac{2c_2}{\pi} f_2 h^{-2} + \frac{c_3}{\pi} f_3 h^{-3}, \tag{3.3}$$

where the functions $f_i = f_i(x,y)$ are determined by solving the following boundary-value problems in a fixed region D^0:

$$\Delta f_i = 0, \quad (f_i)_{y=0} = 0, \quad \Big(\frac{\partial f_i}{\partial n}\Big)_{S_3^0} = (Q_i)_{S_3^0}, \quad i = 1,2,3.$$

After the inverse variable substitution $x \to \varepsilon x$, $y \to \varepsilon y$, and $\varepsilon = h^{-1}$ in (3.3), the functions $f_i(\varepsilon x, \varepsilon y)$ are expanded in a Taylor series with center at the point $\varepsilon = 0$ ($h = \infty$):

$$f_i(\varepsilon x, \varepsilon y) = -\xi_i y\varepsilon + \eta_i xy\varepsilon^2 + \mu_i(3x^2y - y^3)\varepsilon^3/6 + \ldots, \quad i = 1,2,3,$$

$$\xi_i = -f_{iy}, \quad \eta_i = f_{ixy}, \quad \mu_i = f_{ixxy} = -f_{iyyy}, \quad i = 1,2,3,$$

where the partial derivatives are calculated at the point $M_0 = (0,0)$. As a result, we obtain the following asymptotic form of the potential Φ_2, which is valid in any fixed (independent of h) neighborhood of the plate:

$$\Phi_2(x,y) = -\frac{c_1\xi_1}{\pi} yh^{-2} - \Big[\frac{2c_2\xi_2}{\pi} y - \frac{c_1\eta_1}{\pi} xy\Big]h^{-3} - \Big[\frac{c_3\xi_3}{\pi} y - \frac{2c_2\eta_2}{\pi} xy - \frac{c_1\mu_1}{6\pi}(3x^2y - y^3)\Big]h^{-4} + \ldots\ .$$

After this, we must eliminate the errors caused by the potential Φ_2 on the plate surface. For the function $u = \Phi_1 + \Phi_3$, the corresponding problem is formulated as

$$\Delta u = 0, \quad r \in G, \quad (u)_{S_2} = 0, \quad (u)_\infty = 0; \tag{3.4}$$

$$\frac{\partial u}{\partial y} - g \geqslant 0, \quad u\Big(\frac{\partial u}{\partial y} - g\Big) = 0, \quad u \leqslant 0, \qquad y = 0,\ |x| < a, \qquad g = g(x) = v_1 - \omega_1 x - kx^2,$$

$$v_1 = v_0 + \frac{c_1\xi_1}{\pi} h^{-2} + \frac{2c_2\xi_2}{\pi} h^{-3} + \frac{c_3\xi_3}{\pi} h^{-4}, \quad \omega_1 = \omega + \frac{c_1\eta_1}{\pi} h^{-3} + \frac{2c_2\eta_2}{\pi} h^{-4}, \quad k = \frac{c_1\mu_1}{2\pi} h^{-4}.$$

The function u on the plate surface and the point corresponding to this function $c_u = \min\limits_c\{c\ (u(x,0) = 0,$ $c < x < a\})$ are the second approximations of the velocity potential Φ on S_1 and the separation point c, respectively; Φ_1 and c_∞ were used as first approximations.

At this stage, if we confine ourselves to terms of order h^{-3} inclusive, the problem for u differs from the problem for the potential Φ_1 only in velocities. In this case, the function u on the plate surface and the point c_u are determined from formulas (2.7) and (2.8), in which Φ_1 and c_∞ are replaced by u and c_u and v_0 and ω are replaced by v_1 and ω_1, respectively. Expanding the above formulas in power series in h^{-1} and retaining terms of order h^{-3}, we obtain the desired asymptotic expressions. However, for the further consideration, the next term of order h^{-4} must be taken into account.

Problem (3.4) with the quadratic function g is solved similarly to the problem for Φ_1: for any fixed c on the segment $[-a, a]$, one first construct a solution of the mixed boundary-value problem with a half-plane, whose boundary $y = 0$ has a segment $[-a, c]$ separating the first-type and second-type boundary conditions; the point c_u is then determined from two conditions written as inequalities. The relations for the point c_u and the function u on the plate surface are finally written as

$$c_u = \frac{4v_1}{3\omega_1} + \frac{a}{3} - \frac{4k}{27\omega_1}\Big[10\Big(\frac{v_1}{\omega_1}\Big)^2 + 2\frac{v_1}{\omega_1} a + a^2\Big] + O(k^2), \qquad k \to 0,$$

$$u(x,0) = -\sqrt{\Big(\frac{a+c_u}{2}\Big)^2 - t^2}\,\Big[v_1 + \omega_1 \frac{a-c_u}{2} - k\Big(\frac{a-c_u}{2}\Big)^2 \tag{3.5}$$

$$- (\omega_1 - k(a-c_u))\frac{t}{2} - \frac{k}{6}\Big(\Big(\frac{a+c_u}{2}\Big)^2 + 2t^2\Big)\Big], \qquad t = x + \frac{a-c_u}{2}.$$

The approximate solutions obtained above need to be refined because the next approximations of the velocity potential Φ on S_1 and the separation point c also contain terms of order h^{-4}. Therefore, it is necessary to continue the process of constructing successive approximations and consider the boundary-value problems in the regions D and G. Determining the asymptotic form of the potential $\Phi_3 = u - \Phi_1$ at a large distance from the plate

$$\Phi_3 = -\frac{h^{-2}c_4 y}{\pi(x^2+y^2)} - \ldots, \qquad c_4 = 8^{-1}(a+c)^2 c_1 \xi_1,$$

we formulate the problem of eliminating the discrepancies caused by the function Φ_3 on the immovable boundary S_3 (problem for the potential Φ_4). The asymptotic form of the function Φ_4 in the vicinity of the plate is derived similarly to the asymptotic form of the potential Φ_2:

$$\Phi_4 = -\pi^{-1} c_4 \xi_1 y h^{-4} - \ldots \, .$$

After this, we eliminate the discrepancies caused by the function Φ_4 on the plate surface. The problem for eliminating the discrepancies on S_1 has the form of (3.4), where the functions u and g are replaced by $v = \Phi_1 + \Phi_3 + \Phi_5$ and $g_1 = g + (c_4\xi_1/\pi)h^{-4}$, respectively. As a result, the refined point c_u and the function u on S_1 are determined from formulas (3.5), in which the term $(c_4\xi_1/\pi)h^{-4}$ is added to the relation for v_1. Thus, we find the third approximation of the solution of the initial problem. We note that the next approximations add terms of higher order smallness than h^{-4}.

Several elementary transformations for the separation point c yield the asymptotic formula

$$c = c_\infty + \frac{1}{3}\Big(\frac{a+c_\infty}{2}\Big)^3 (\xi_1 h^{-2} - p h^{-3} + q h^{-4}) + O(h^{-5}), \qquad h \to \infty,$$

$$p = [(5a - 3c_\infty)\xi_2 + (3c_\infty - a)\eta_1]/4,$$

$$16q = (7a^2 - 6ac_\infty + 3c_\infty^2)\xi_3 + 2(a+c_\infty)^2\xi_1^2 + (3c_\infty - a)(5a - 3c_\infty)\eta_2 - (a^2 - 2ac_\infty + 5c_\infty^2)\mu_1,$$

where c_∞ is related to the velocities v_0 and ω by formula (2.7).

To simplify the following representations, we assume that the region D is symmetric with respect to the axis y. In this case, $\xi_2 = 0$ and $\eta_1 = 0$, and, hence, $p = 0$. The asymptotic expressions for the momentum I, the angular moment M, and the momentum point x_0 are written as

$$\begin{aligned} I &= -\rho\omega_1 \frac{\pi}{4}\Big(\frac{a+c}{2}\Big)^3\Big[1 + \frac{\mu_1}{256}(a+c)^3(5c - 3a)h^{-4}\Big] + O(h^{-5}), \quad h \to \infty, \\ M &= -\rho\omega_1 \frac{\pi}{32}\Big(\frac{a+c}{2}\Big)^3\Big[5a - 3c - \frac{\mu_1}{16}(a-c)^2(a+c)^3 h^{-4}\Big] + O(h^{-5}), \quad h \to \infty; \end{aligned} \tag{3.6}$$

$$x_0 = -\frac{5a - 3c}{8} + \frac{\mu_1}{2048}(a+c)^5 h^{-4} + O(h^{-5}), \quad h \to \infty. \tag{3.7}$$

More detailed asymptotic formulas for I, M, and x_0 are derived by substituting the relations for ω_1 and c into (3.6) and (3.7), respectively, with subsequent series expansion in powers of h^{-1}. The asymptotic coefficients obtained in such a manner are expressed in terms of the velocities v_0 and ω, which are assumed to be independent of h.

In the impact problem, the specified parameters are obviously the external impact momentum P and its point x_0. In this case, the impact-induced plate velocities v_0 and ω are determined from the system of nonlinear equations (1.9). We note that the separation point c depends only on the momentum point x_0. To determine this point, we obtain a nonlinear equation in the form of (3.7) for large h. Fixing x_0 and solving this equation asymptotically for c, we have

$$c = c_\infty - \mu_1(a + c_\infty)^5 h^{-4}/768 + O(h^{-5}), \qquad h \to \infty, \tag{3.8}$$

where c_∞ is related to x_0 by formula (2.9).

4. Appendices. We assume that in the case of both an unbounded fluid and a bounded fluid, the separation impact on the plate is induced by the action of an external impact momentum applied at the point $(x_0, 0)$. Then, for sufficiently large h, formula (3.8) allows one to conclude on the effect of the tank walls of arbitrary shape on the zone of fluid particle separation from the plate surface. The problem reduces to determining the sign of the coefficient μ_1, which depends only on the shape of the tank walls. For $\mu_1 > 0$, the separation zone increases, and for $\mu_1 < 0$, it decreases compared with the case of an unbounded fluid. For $\mu_1 = 0$, the question remains open. Examples describing all possible cases are given below:

— for a horizontal layer,

$$\mu_1 = 21\pi^4/(2880b^4) > 0;$$

— for a vertical layer,

$$\mu_1 = -\pi^4/(120b^4) < 0;$$

— for a truncated circular lune,

$$\mu_1 = \frac{8}{c^4}\int\limits_0^\infty \frac{\lambda(1-2\lambda^2)\cosh{(\pi-\beta_0)\lambda}\,d\lambda}{\sinh{\pi\lambda}\cosh{\beta_0\lambda}}, \qquad b = -c\cot{\beta_0}, \quad 0 < \beta_0 < \pi.$$

Here b is the characteristic size of the region D^0. In the first example, this is the depth of the fixed layer, in the second case, this is half the distance between the vertical walls, and in the third case, the coordinate of the center of the lune arc.

The truncated circular lune is the region bounded by a straight-line segment ($y = 0$, $-c < x < c$) and the circular arc passing through the points $x = \pm c$. For $b < 0$ ($b > 0$), the constant $\mu_1 < 0$ ($\mu_1 > 0$), and the sign of μ_1 changes with passage through a semicircle (tank shape is a half-cylinder) for which $\mu_1 = 0$.

Thus, a tank shaped like a half-cylinder has little effect on the zone of fluid particle separation from the plate surface.

Conclusions. An algorithm for constructing asymptotic solutions for linear problems of hydrodynamic impact for large h was proposed in [6–8]. In the present paper, this algorithm is extended to a nonlinear problem. The asymptotic expressions obtained were used to study the effect of tank walls of various shapes on the zone of fluid particle separation from the plate surface. It has been established that depending on the tank walls, this zone can either increase or decrease compared with the case of an unbounded fluid.

The proposed asymptotic approach can be employed to solve other mixed problems of mathematical physics with an *a priori* unknown contact area.

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